Lecture Notes, May 9, 2011 --- Part 1

General Equilibrium in an Economy with unbounded technology sets

Delete P.VI (bounded $\mathcal{Y}^j$). Like all good mathematicians, we're reducing this to the previous case.

Under assumptions of No Free Lunch (P.IV(a)) and Irreversibility (P.IV(b)), the attainable output set for the economy and for each firm is still bounded.

P.IV. (a) if $y \in Y$ and $y \neq 0$, then $y_k < 0$ for some k.
(b) if $.y \in Y$ and $y \neq 0$, then $-y \notin Y$

Let firm j's (unbounded) production technology be Y^j. Define $S^j(p)$ as j's profit maximizing supply in Y^j. Define $D^i(p)$ as i's demand without restriction to $\{x| |x|\leq c\}$ and with income $M^i(p)= p \cdot r^i + \Sigma_j \alpha^{ij}\pi^j(p)$. Note that $S^j(p)$ and $D^i(p)$ may not be well defined.

Define $\widetilde{Y}^j = Y^j \cap \{x| |x|\leq c\}$, substitute $\widetilde{Y}^j$ for $\mathcal{Y}^j$ in chapters 11 - 14. Define $\widetilde{S}^j(p)$ as j's supply function based on $\widetilde{Y}^j$.

Theorem 15.3(b): If $\widetilde{S}^j(p)$ is attainable, then $S^j(p) = \widetilde{S}^j(p)$.

Theorem 16.1(b): If $M^i(p) = \widetilde{M}^i(p)$, and $\widetilde{D}^i(p)$ is attainable, then $\widetilde{D}^i(p) = D^i(p)$.

$Z(p) = \Sigma_i D^i(p) - \Sigma_j S^j(p) - \Sigma_i r^i$

Theorem 18.1: Assume P.II-P.V, and C.I-C.V, C.VI(SC), CVII. There is $p^* \in P$ so that p^* is an equilibrium price vector. That is, $Z(p^*) \leq 0$ and $p^*_k = 0$ for k so that $Z_k(p^*) < 0$.

Proof: The artificially bounded economy characterized by production technologies $\widetilde{Y}^j$, $j \in F$, is a special case of the bounded economy of chapters 11 - 14. Find equilibrium of that bounded economy. That bounded economy equilibrium is attainable so restriction to length c is not a binding constraint. So bounded and unbounded supply and demand coincide. Equilibrium prices of the bounded

economy exist and are equilibrium prices for the unbounded economy with technology sets Y^j. Q.E.D.

Theorem 18.1 here is the most important single result of this course. It says that the competitive economy, guided only by prices, has a market clearing equilibrium outcome. The decentralized price-guided economy has a consistent solution. This is the defining result of the general equilibrium theory.